FFT solver non-linear problems

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Impact

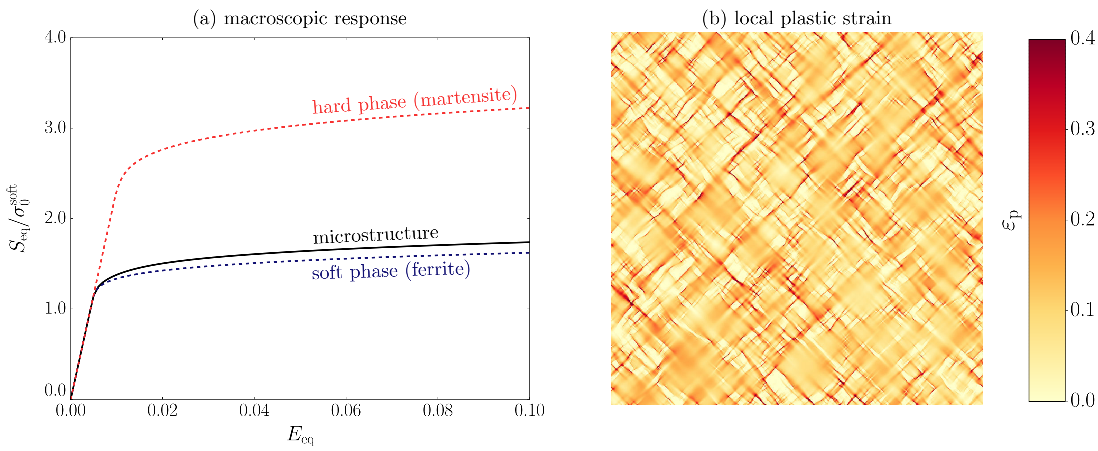
Computational micromechanics and homogenization require the solution of the mechanical equilibrium of a periodic cell that comprises a (generally complex) microstructure. Techniques that apply the Fast Fourier Transform have attracted much attention as they outperform other methods in terms of speed and memory footprint. Moreover, the Fast Fourier Transform is a natural companion of pixel-based digital images which often serve as input. In its original form, one of the biggest challenges for the method is the treatment of (geometrically) non-linear problems, partially due to the need for a uniform linear reference problem.

Variational formulation

In a geometrically linear setting, we treat the problem in a variational form resulting in an unconditionally stable scheme that combines Newton iterations with an iterative linear solver, and therefore exhibits robust and quadratic convergence behaviour. Through this approach, well-known key ingredients were recovered in terms of discretization, numerical quadrature, consistent linearization of the material model, and the iterative solution of the resulting linear system.

Geometric non-linearities

The extension to finite strains, using arbitrary constitutive models, is now straightforward. Moreover, because of the application of the Fast Fourier Transform, the implementation is substantially easier than that of other (Finite Element) methods. Both claims are demonstrated with a ‘simple code in Python of just 59 lines (without comments).



Selection of publications

[1] T.W.J. de Geus, J. Vondřejc, J. Zeman, R.H.J. Peerlings, and M.G.D. Geers. Finite strain FFT-based non-linear solvers made simple. *Comput. Methods Appl. Mech. Eng.*, 318:412–430, 2017. doi: 10.1016/j.cma.2016.12.032. arXiv: 1603.08893.

[2] J. Zeman, T.W.J. de Geus, J. Vondřejc, R.H.J. Peerlings, and M.G.D. Geers. A finite element perspective on nonlinear FFT-based micromechanical simulations. *Int. J. Numer. Methods Eng.*, 111(10):903–926, 2017. doi: 10.1002/nme.5481. arXiv: 1601.05970.